

## Boolsche Algebra

Distributivität

$$\begin{aligned}
 a \cdot (b + c) &= (a \cdot b) + (a \cdot c) \\
 a + (b \cdot c) &= (a + b) \cdot (a + c) \\
 a \cdot (b \oplus c) &= (a \cdot b) \oplus (a \cdot c) \\
 a + (b \equiv c) &= (a + b) \equiv (a + c)
 \end{aligned}$$

de Morgansche Regeln

$$\begin{aligned}
 -(a \cdot b) &= -a + -b \\
 -(a + b) &= -a \cdot -b
 \end{aligned}$$

Umwandlung in Grundfunktionen

$$\begin{aligned}
 a \equiv b &= a \oplus b = (a \cdot b) + (-a \cdot -b) = (a + b)(-a + b) \\
 a \oplus b &= a \equiv -b = (a \cdot -b) + (-a \cdot b) = (a + b)(-a + b) \\
 a \rightarrow b &= -a + b = -(a \cdot -b)
 \end{aligned}$$

Literal  $a, -a$   
 Minterm  $a_1 a_2 \dots$   
 Maxterm  $a_1 + a_2 + \dots$   
 Konjunktion  $\cdot$   
 Disjunktion  $+$

disjunktive Normalform: Disjunktion von Mintermen  
 konjunktive Normalform: Konjunktion von Maxtermen  
 ausgezeichnete Normalform: Jeder Term enthält alle Literale

$$a \oplus b \oplus c = a \equiv b \equiv c = (a \cdot b \cdot c) + (-a \cdot -b \cdot c) + (-a \cdot b \cdot -c) + (a \cdot -b \cdot -c)$$

Komplement

$$\begin{aligned}
 a + -a &= 1 & a \oplus -a &= 1 \\
 a \cdot -a &= 0 & a \equiv -a &= 0
 \end{aligned}$$

Absorptionsregeln

$$\begin{aligned}
 a + (a \cdot b) &= a \\
 a \cdot (a + b) &= a
 \end{aligned}$$

## Automaten (Zustand u, Eingang x, Ausgang y)

Medwedjew-Automat  $u := f(u, x) \quad y = u$   
 Moore-Automat  $u := f(u, x) \quad y = g(u)$   
 Mealy-Automat  $u := f(u, x) \quad y = g(u, x)$

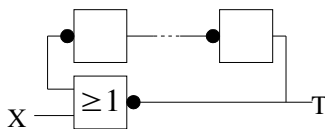
Kodierer: Überträgt den gesetzten Eingang in Binärzahl  
 Dekodierer: Setzt einen gewählten Ausgang  
 Multiplexer: Schaltet den gewählten Eingang auf den Ausgang durch  
 Demultiplexer: Schaltet den Eingang auf den gewählten Ausgang durch

### Addierer

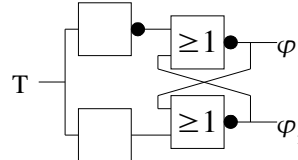
$$\begin{aligned}
 U &= ab + au + bu & S &= a \oplus b \oplus u \\
 c &= u_n, n = Z_{n-1}, v = u_n \oplus u_{n-1}, z = -Z_{n-1} \cdot \dots \cdot -Z_2 \cdot -Z_1 \cdot -Z_0
 \end{aligned}$$

## Takt

Takterzeugung



2-Phasen-Takt



## Hazards

Logischer Test

NOT	
0	1
↓	↑
↑	↓
1	0

AND	0	↓	↑	1
0	0	0	0	0
↓	0	↓	↑	↓
↑	0	↑	↓	↑
1	0	↓	↑	1

OR	0	↓	↑	1
0	0	↓	↑	1
↓	↓	↓	↑	1
↑	↑	↓	↑	1
1	1	1	1	1

Technischer Test

$$\begin{aligned}
 \downarrow^t \cdot \uparrow^{t-n} &= 0 \\
 \uparrow^t + \downarrow^{t-n} &= 1 \\
 \downarrow^t \cdot \downarrow^{t-n} &= \downarrow^t \\
 \uparrow^t \cdot \uparrow^{t-n} &= \uparrow^t \\
 \uparrow^t + \uparrow^{t-n} &= \uparrow^t \\
 \downarrow^t + \downarrow^{t-n} &= \downarrow^{t-n}
 \end{aligned}$$

Hazards:

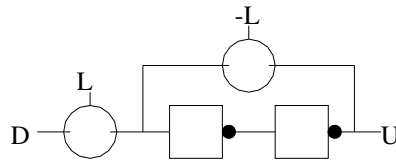
$$\begin{aligned}
 \uparrow^t \cdot \downarrow^{t-n} \\
 \downarrow^t + \uparrow^{t-n} \\
 \downarrow^t \cdot \uparrow^t = \uparrow^t \cdot \downarrow^t = \downarrow^t \\
 \uparrow^t + \downarrow^t = \downarrow^t + \uparrow^t = \downarrow^t
 \end{aligned}$$

# FlipFlops

## DL-Flipflop

$$U := D \cdot L + U \cdot \bar{L}$$

D	L	U
-	0	U
0	1	0
1	1	1

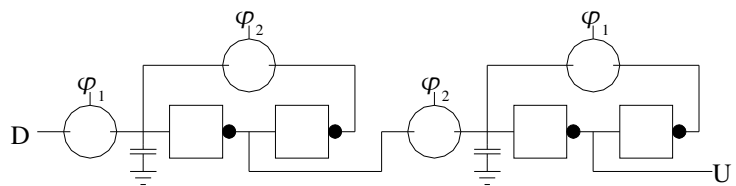
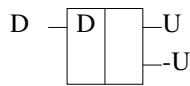
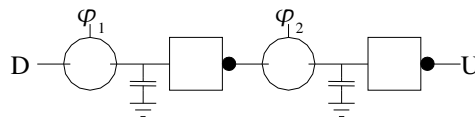


## D-Flipflop

$$U := D$$

$$D = U^d = S + \bar{R} \cdot U = J \cdot \bar{U} + \bar{K} \cdot U$$

D	U
0	0
1	1



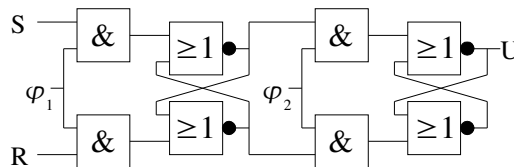
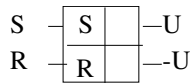
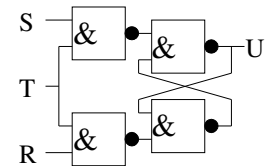
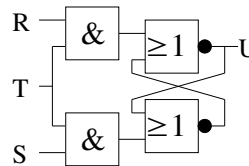
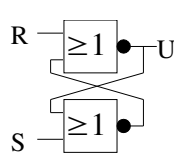
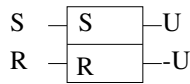
## SR-Flipflop

$$U := S + \bar{R} \cdot U$$

$$S = U^d|_{U=0} [\cdot \bar{U}] = D = J \cdot \bar{U}$$

$$R = \bar{U}^d|_{U=1} [\cdot U] = \bar{D} = K \cdot U$$

S	R	U
0	0	U
0	1	0
1	0	1
1	1	!!!



## JK-Flipflop

$$U := J \cdot \bar{U} + \bar{K} \cdot U$$

$$J = U^d|_{U=0} = D = S$$

$$K = \bar{U}^d|_{U=1} = \bar{D} = R$$

J	K	U
0	0	U
0	1	0
1	0	1
1	1	-U

