

**Zustandsmodell**

kontinuierlich

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{m}(t), t) \quad \text{Zustand}$$

$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{n}(t), t) \quad \text{Messung}$$

Signale

$$\mathbf{u} \sim \mathcal{N}(\tilde{\mathbf{u}}, \mathbf{U} \quad [tu^2]) \quad \text{Systemeingang}$$

$$\mathbf{m} \sim \mathcal{N}(\mathbf{0}, \mathbf{M} \quad [tm^2]) \quad \text{Prozeßrauschen}$$

$$\mathbf{n} \sim \mathcal{N}(\mathbf{0}, \mathbf{N} \quad [tn^2]) \quad \text{Meßrauschen}$$

diskret

$$\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{m}_k) = \int_{t_k}^{t_{k+1}} \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{m}(t), t) dt + \mathbf{x}_k \approx T\mathbf{f}(\mathbf{x}(t_k), \mathbf{u}(t_k), \mathbf{m}(t_k), t_k) + \mathbf{x}_k$$

$$\mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{n}_k) = \mathbf{h}(\mathbf{x}(t_k), \mathbf{u}(t_k), \mathbf{m}(t_k), t_k)$$

$$\mathbf{u}_k \sim \mathcal{N}(\tilde{\mathbf{u}}_k, \mathbf{U}_k \quad [u_k^2])$$

$$\mathbf{m}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{M}_k \quad [m_k^2])$$

$$\mathbf{n}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{N}_k \quad [n_k^2])$$

$$\mathbf{B}_k \cdot \mathbf{U}_k \cdot \mathbf{B}_k^T = \int_{t_k}^{t_{k+1}} \mathbf{A}'(t, t_{k+1}) \cdot \mathbf{B} \cdot \mathbf{U} \cdot \mathbf{B}^T \cdot \mathbf{A}'(t, t_{k+1})^T dt$$

$$\mathbf{E}_k \cdot \mathbf{M}_k \cdot \mathbf{E}_k^T = \int_{t_k}^{t_{k+1}} \mathbf{A}'(t, t_{k+1}) \cdot \mathbf{E} \cdot \mathbf{M} \cdot \mathbf{E}^T \cdot \mathbf{A}'(t, t_{k+1})^T dt$$

$$\mathbf{D}_k \cdot \mathbf{U}_k \cdot \mathbf{D}_k^T = \int_{t_k}^{t_{k+1}} \mathbf{D} \cdot \mathbf{U} \cdot \mathbf{D}^T dt$$

$$\mathbf{F}_k \cdot \mathbf{N}_k \cdot \mathbf{F}_k^T = \int_{t_k}^{t_{k+1}} \mathbf{F} \cdot \mathbf{N} \cdot \mathbf{F}^T dt$$

$$\mathbf{u}_k = \tilde{\mathbf{u}} \quad | \approx T \mathbf{U}$$

$$\mathbf{m}_k = \mathbf{m} \quad | \approx T \mathbf{M}$$

$$\mathbf{n}_k = \mathbf{n} \quad | \approx T \mathbf{N}$$

$$\mathbf{U}_k \approx \frac{1}{T} \mathbf{U} \quad | \approx T \mathbf{U}$$

$$\mathbf{M}_k \approx \frac{1}{T} \mathbf{M} \quad | \approx T \mathbf{M}$$

$$\mathbf{N}_k \approx \frac{1}{T} \mathbf{N} \quad | \approx T \mathbf{N}$$

Linearisierung (EKF)

$$\dot{\mathbf{x}}(t) = \mathbf{A} \cdot \mathbf{x} + \mathbf{B} \cdot \mathbf{u} + \mathbf{E} \cdot \mathbf{m} + \dot{\mathbf{x}}_f \quad \left[ \frac{x}{t} \right]$$

$$\mathbf{y}(t) = \mathbf{C} \cdot \mathbf{x} + \mathbf{D} \cdot \mathbf{u} + \mathbf{F} \cdot \mathbf{n} + \mathbf{y}_h \quad [y]$$

$$\dot{\mathbf{x}}_f(t) = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t) - \mathbf{A} \cdot \mathbf{x} - \mathbf{B} \cdot \mathbf{u} \quad \left[ \frac{x}{t} \right]$$

$$\mathbf{y}_h(t) = \mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t) - \mathbf{C} \cdot \mathbf{x} - \mathbf{D} \cdot \mathbf{u} \quad [y]$$

$$\mathbf{A}(t) = \frac{\partial \mathbf{f}}{\partial \mathbf{x}^T}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t) \quad \left[ \frac{1}{t} \right]$$

$$\mathbf{B}(t) = \frac{\partial \mathbf{f}}{\partial \mathbf{u}^T}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t) \quad \left[ \frac{x}{tu} \right]$$

$$\mathbf{E}(t) = \frac{\partial \mathbf{f}}{\partial \mathbf{m}^T}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t) \quad \left[ \frac{x}{tm} \right]$$

$$\mathbf{C}(t) = \frac{\partial \mathbf{h}}{\partial \mathbf{x}^T}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t) \quad \left[ \frac{y}{x} \right]$$

$$\mathbf{D}(t) = \frac{\partial \mathbf{h}}{\partial \mathbf{u}^T}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t) \quad \left[ \frac{y}{u} \right]$$

$$\mathbf{F}(t) = \frac{\partial \mathbf{h}}{\partial \mathbf{n}^T}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t) \quad \left[ \frac{y}{n} \right]$$

$$\mathbf{x}_{k+1} = \mathbf{A}_k \cdot \mathbf{x}_k + \mathbf{B}_k \cdot \mathbf{u}_k + \mathbf{E}_k \cdot \mathbf{m}_k + \mathbf{x}_{fk}$$

$$\mathbf{y}_k = \mathbf{C}_k \cdot \mathbf{x}_k + \mathbf{D}_k \cdot \mathbf{u}_k + \mathbf{F}_k \cdot \mathbf{n}_k + \mathbf{y}_{hk}$$

$$\mathbf{x}_{fk} = \mathbf{f}_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{0}) - \mathbf{A}_k \cdot \mathbf{x}_k - \mathbf{B}_k \cdot \mathbf{u}_k$$

$$\mathbf{y}_{hk} = \mathbf{h}_k(\mathbf{x}_k^+, \mathbf{u}_{k+1}, \mathbf{0}) - \mathbf{C}_k \cdot \mathbf{x}_k - \mathbf{D}_k \cdot \mathbf{u}_k$$

$$\mathbf{A}_k = \frac{\partial \mathbf{f}_k}{\partial \mathbf{x}^T}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{0}) \quad \mathbf{A}_k = \mathbf{A}'(t_k, t_{k+1}) := \exp\left(\int_{t_k}^{t_{k+1}} \mathbf{A} dt\right) \quad \mathbf{A}_k \approx T\mathbf{A} + \mathbf{1} \quad [1]$$

$$\mathbf{B}_k = \frac{\partial \mathbf{f}_k}{\partial \mathbf{u}^T}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{0}) \quad \mathbf{B}_k \cdot \mathbf{u}_k = \int_{t_k}^{t_{k+1}} \mathbf{A}'(t, t_{k+1}) \cdot \mathbf{B} \cdot \mathbf{u} dt \quad \mathbf{B}_k \approx T\mathbf{B} \quad | = \mathbf{B} \quad \left[ \frac{x}{u_k} \right]$$

$$\mathbf{E}_k = \frac{\partial \mathbf{f}_k}{\partial \mathbf{m}^T}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{0}) \quad \mathbf{E}_k \cdot \mathbf{m}_k = \int_{t_k}^{t_{k+1}} \mathbf{A}'(t, t_{k+1}) \cdot \mathbf{E} \cdot \mathbf{m} dt \quad \mathbf{E}_k \approx T\mathbf{E} \quad | = \mathbf{E} \quad \left[ \frac{x}{m_k} \right]$$

$$\mathbf{C}_k^+ = \frac{\partial \mathbf{h}_k}{\partial \mathbf{x}^T}(\mathbf{x}_k^+, \mathbf{u}_{k+1}, \mathbf{0}) \quad \mathbf{C}_k^+ = \mathbf{C} \quad \left[ \frac{y}{x} \right]$$

$$\mathbf{D}_k^+ = \frac{\partial \mathbf{h}_k}{\partial \mathbf{u}^T}(\mathbf{x}_k^+, \mathbf{u}_{k+1}, \mathbf{0}) \quad \mathbf{D}_k \cdot \mathbf{u}_k = \mathbf{D} \cdot \mathbf{u} \quad \mathbf{D}_k^+ = \mathbf{D} \quad | \approx \frac{1}{T} \mathbf{D} \quad \left[ \frac{y}{u_k} \right]$$

$$\mathbf{F}_k^+ = \frac{\partial \mathbf{h}_k}{\partial \mathbf{n}^T}(\mathbf{x}_k^+, \mathbf{u}_{k+1}, \mathbf{0}) \quad \mathbf{F}_k \cdot \mathbf{n}_k = \mathbf{F} \cdot \mathbf{n} \quad \mathbf{F}_k^+ = \mathbf{F} \quad | \approx \frac{1}{T} \mathbf{F} \quad \left[ \frac{y}{n_k} \right]$$

**Kalman-Bucy-Filter**

$$\dot{\mathbf{x}}^+ = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t) \quad \left[ \frac{x}{t} \right]$$

$$\mathbf{y}^+ = \mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t) \quad [y]$$

$$\dot{\mathbf{X}}^+ = \mathbf{A} \cdot \mathbf{X} + \mathbf{X} \cdot \mathbf{A}^T + \mathbf{B} \cdot \mathbf{U} \cdot \mathbf{B}^T + \mathbf{E} \cdot \mathbf{M} \cdot \mathbf{E}^T \quad \left[ \frac{x^2}{t} \right]$$

$$\mathbf{K} = \mathbf{X} \cdot \mathbf{C}^T \cdot (\mathbf{F} \cdot \mathbf{N} \cdot \mathbf{F}^T + \mathbf{D} \cdot \mathbf{U} \cdot \mathbf{D}^T)^{-1} \quad \left[ \frac{x}{ty} \right]$$

$$\dot{\mathbf{x}} = \dot{\mathbf{x}}^+ + \mathbf{K} \cdot (\mathbf{y} - \mathbf{y}^+) \quad \left[ \frac{x}{t} \right]$$

$$\dot{\mathbf{X}} = \dot{\mathbf{X}}^+ - \mathbf{K} \cdot \mathbf{C} \cdot \mathbf{X} \quad \left[ \frac{x^2}{t} \right]$$

**Kalman-Filter**

$$\mathbf{x}_{k+1}^+ = \mathbf{f}_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{0}) \quad \text{Zustandsvorhersage} \quad [x]$$

$$\mathbf{y}_{k+1}^+ = \mathbf{h}_{k+1}(\mathbf{x}_{k+1}^+, \mathbf{u}_{k+1}, \mathbf{0}) \quad \text{Meßvorhersage} \quad [y]$$

$$\mathbf{X}_{k+1}^+ = \mathbf{A}_k \cdot \mathbf{X}_k \cdot \mathbf{A}_k^T + \mathbf{B}_k \cdot \mathbf{U}_k \cdot \mathbf{B}_k^T + \mathbf{E}_k \cdot \mathbf{M}_k \cdot \mathbf{E}_k^T \quad \text{Vorhersagefehler} \quad [x^2]$$

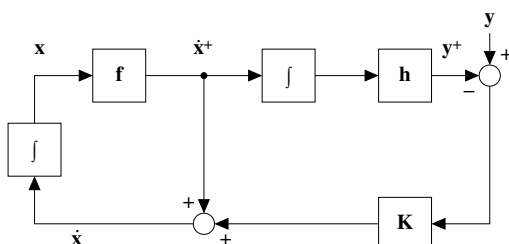
$$\mathbf{Y}_{k+1}^+ = \mathbf{C}_{k+1}^+ \cdot \mathbf{X}_{k+1}^+ \cdot \mathbf{C}_{k+1}^{+T} + \mathbf{D}_{k+1}^+ \cdot \mathbf{U}_{k+1} \cdot \mathbf{D}_{k+1}^{+T} \quad \text{Meßvorhersagefehler} \quad [y^2]$$

$$\mathbf{K}_{k+1} = \mathbf{X}_{k+1}^+ \cdot \mathbf{C}_{k+1}^{+T} \cdot (\mathbf{F}_{k+1}^+ \cdot \mathbf{N}_{k+1} \cdot \mathbf{F}_{k+1}^{+T} + \mathbf{Y}_{k+1}^+)^{-1} \quad \text{Kalman-Gain} \quad \left[ \frac{x}{y} \right]$$

$$\mathbf{x}_{k+1} = \mathbf{x}_{k+1}^+ + \mathbf{K}_{k+1} \cdot (\mathbf{y}_{k+1} - \mathbf{y}_{k+1}^+) \quad \text{Neuer Zustand} \quad [x]$$

$$\mathbf{X}_{k+1} = (\mathbf{1} - \mathbf{K}_{k+1} \cdot \mathbf{C}_{k+1}^+) \cdot \mathbf{X}_{k+1}^+ \quad \text{Zustandsvarianz} \quad [x^2]$$

$$= (\mathbf{1} - \mathbf{K}_{k+1} \cdot \mathbf{C}_{k+1}^+) \cdot \mathbf{X}_{k+1}^+ \cdot (\mathbf{1} - \mathbf{K}_{k+1} \cdot \mathbf{C}_{k+1}^+)^T + \mathbf{K}_{k+1} \cdot \mathbf{F}_{k+1}^+ \cdot \mathbf{N}_{k+1} \cdot \mathbf{F}_{k+1}^{+T} \cdot \mathbf{K}_{k+1}^T \quad \text{Joseph-Form}$$



Informationsfilter (mit  $\mathbf{D} = \mathbf{0}$ )

$$\mathbf{X}' := \mathbf{X}^{-1}$$

$$\mathbf{x}' := \mathbf{X}' \cdot \mathbf{x}$$

$$\mathbf{X}_{k+1}' = \mathbf{X}_{k+1}^{+1}$$

$$\mathbf{x}_{k+1}' = \mathbf{X}_{k+1}' \cdot \mathbf{x}_{k+1}^+$$

$$\mathbf{X}_{k+1}' = \mathbf{X}_{k+1}' + \mathbf{C}_{k+1}^{+T} \cdot \mathbf{F}_{k+1}^{+T-1} \cdot \mathbf{N}_{k+1}^{-1} \cdot \mathbf{F}_{k+1}^{+1} \cdot \mathbf{C}_{k+1}^+$$

$$\mathbf{x}_{k+1}' = \mathbf{x}_{k+1}' + \mathbf{C}_{k+1}^{+T} \cdot \mathbf{F}_{k+1}^{+T-1} \cdot \mathbf{N}_{k+1}^{-1} \cdot \mathbf{F}_{k+1}^{+1} \cdot (\mathbf{y}_{k+1} - \mathbf{y}_{k+1}^+ + \mathbf{C}_{k+1}^+ \cdot \mathbf{x}_{k+1}^+)$$

