Elementarsignale

Signumfunktion

$$\mathrm{sgn}(t)=\begin{cases}-1 & \text{für } t<0\\ 0 & \text{für } t=0\\ 1 & \text{für } t>0\end{cases}=2\sigma(t)-1=2\int_{-\infty}^{t}\delta(\tau)\mathrm{d}\tau-1$$

$$\mathrm{sgn}(n)=\begin{cases}-1 & \text{für } n<0\\ 1 & \text{für } n\geq 0\end{cases}=2\sigma(n)-1=2\sum_{\nu=-\infty}^{n}\delta(\nu)-1$$

$\mathcal{F}(\mathrm{sgn}(t))=\frac{2}{\mathrm{i}\omega}$ $\quad\mathcal{L}(\mathrm{sgn}(t))=\frac{2}{\mathrm{s}}$

Sprungfunktion

$$\sigma(t)=\begin{cases}0 & \text{für } t<0\\ 0{,}5 & \text{für } t=0\\ 1 & \text{für } t>0\end{cases}=\frac{1}{2}\mathrm{sgn}(t)+\frac{1}{2}=\int_{-\infty}^{t}\delta(\tau)\mathrm{d}\tau=\int_{0}^{\infty}\delta(t-\tau)\mathrm{d}\tau$$

$$\sigma(n)=\begin{cases}0 & \text{für } n<0\\ 1 & \text{für } n\geq 0\end{cases}=\frac{1}{2}\mathrm{sgn}(n)+\frac{1}{2}=\sum_{\nu=-\infty}^{t}\delta(\nu)=\sum_{\nu=0}^{\infty}\delta(n-\nu)$$

$\mathcal{F}(\sigma(t))=\frac{1}{\mathrm{i}\omega}+\pi\delta(\omega)$ $\quad\mathcal{L}(\sigma(t))=\frac{1}{\mathrm{s}}$

Rechteckfunktion

$$\Pi_T(t)=\begin{cases}1 & \text{für } |t|<T/2\\ 0{,}5 & \text{für } |t|=T/2\\ 0 & \text{für } |t|>T/2\end{cases}=\sigma\left(t+\frac{T}{2}\right)-\sigma\left(t-\frac{T}{2}\right)$$

$$\Pi_M(n)=\begin{cases}1 & \text{für } |n|\leq(M-1)/2\\ 0 & \text{für } |n|>(M-1)/2\end{cases}=\sigma\left(n+\frac{M-1}{2}\right)-\sigma\left(n-\frac{M+1}{2}\right)$$

$\mathcal{F}(\Pi_T(t))=T\,\mathrm{si}\left(\frac{\omega T}{2}\right)$

$\mathcal{F}(\Pi_M(n))=\frac{\sin(M\omega T/2)}{\sin(\omega T/2)}$

Dreickfunktion

$$\Lambda_T(t)=\begin{cases}1-|t|/T & \text{für } |t|\leq T\\ 0 & \text{für } |t|>T\end{cases}=\frac{1}{T}\Pi_T(t)\star\Pi_T(t)=\mathrm{r}_T(t+T)-\mathrm{r}_T(t)$$

$$\Lambda_M(n)=\begin{cases}1-|n|/M & \text{für } |n|\leq M\\ 0 & \text{für } |n|>M\end{cases}=\frac{1}{M}\Pi_M(n)\star\Pi_M(n)$$

$\mathcal{F}(\Lambda_T(t))=T\,\mathrm{si}\left(\frac{\omega T}{2}\right)^2$

Rampenfunktion

$$\begin{aligned}\mathrm{r}_T(t)&=\begin{cases}0 & \text{für } t<0\\ t/T & \text{für } 0\leq t\leq T\\ 1 & \text{für } t>T\end{cases}=\frac{1}{T}\int_{-\infty}^{t}\Pi_T\left(\tau-\frac{T}{2}\right)\mathrm{d}\tau\\ &=\frac{t}{T}\Pi_T\left(t-\frac{T}{2}\right)+\sigma(t-T)\end{aligned}$$

$$\mathrm{r}_M(n)=\begin{cases}0 & \text{für } n<0\\ n/M & \text{für } 0\leq n\leq M\\ 1 & \text{für } n>M\end{cases}=\frac{1}{M}\left(\sum_{\nu=-\infty}^{n}\Pi_M\left(\nu-\frac{M}{2}\right)-1\right)$$

$\mathcal{F}(\mathrm{r}_T(t))=\frac{1}{\mathrm{i}\omega}\mathrm{si}\left(\frac{\omega T}{2}\right)\exp\left(-\mathrm{i}\frac{\omega T}{2}\right)+\pi\delta(\omega)$

si-Funktion (Spaltfunktion)

$\mathrm{si}(\omega_0 t)=\frac{\sin(\omega_0 t)}{\omega_0 t}$

$\mathrm{si}(\omega_0 n)=\frac{\sin(\omega_0 nT)}{\omega_0 nT}$

$\mathcal{F}(\mathrm{si}(\omega_T t))=\frac{T}{2}\Pi_{\omega_T}\left(\frac{\omega}{2}\right)$

Deltafunktion

$$\begin{aligned}\delta(t)&=\begin{cases}\infty & \text{für } t=0\\ 0 & \text{für } t\neq 0\end{cases}=\lim_{\varDelta\to 0}\left(\frac{1}{\varDelta}\Pi_\varDelta(t)\right)=\frac{\mathrm{d}\sigma(t)}{\mathrm{d}t}\\ &=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp(\mathrm{i}\omega t)\mathrm{d}\omega=\frac{\Omega}{2\pi}\sum_{k=-\infty}^{\infty}\exp(\mathrm{i}k\Omega t)\end{aligned}$$

$$\begin{aligned}\delta(n)&=\begin{cases}1 & \text{für } n=0\\ 0 & \text{für } n\neq 0\end{cases}=\Pi_1(n)=\sigma(n)-\sigma(n-1)\\ &=\frac{T}{2\pi}\int_{x}^{x+2\pi/T}\exp(\mathrm{i}\omega nT)\mathrm{d}\omega\end{aligned}$$

$\mathcal{F}(\delta(t+t_0))=\exp(\mathrm{i}\omega t_0)$ $\quad\mathcal{L}(\delta(t+t_0))=\exp(\mathrm{s}t_0)$

$\mathcal{F}(\delta(n+n_0))=\exp(\mathrm{i}\omega n_0 T)$ $\quad\mathcal{Z}(\delta(n+n_0))=\mathrm{z}^{n_0}$

$u(t+t_0)=u(t)\star\delta(t+t_0)=\int_{-\infty}^{\infty}u(\tau)\delta(t+t_0-\tau)\mathrm{d}\tau$

$u(n+n_0)=u(n)\star\delta(n+n_0)=\sum_{\nu=-\infty}^{\infty}u(\nu)\delta(n+n_0-\nu)$

Deltakamm

$\delta_T(t)=\sum_{\tau=-\infty}^{\infty}\delta(t-\tau T)$

$\delta_M(n)=\sum_{\nu=-\infty}^{\infty}\delta(n-\nu M)$

$\mathcal{F}(\delta_T(t))=\omega_T\delta_{\omega_T}(\omega)$

$\mathcal{Z}(\delta_M(n))=\sum_{\nu=-\infty}^{\infty}\mathrm{z}^{-\nu M}$

$u_T(t)=u(t)\star\delta_T(t)=\sum_{k=-\infty}^{\infty}u(t-kT)$

Signaleigenschaften

Arithmetischer Mittelwert

$m_u(t_1,t_2) = \frac{1}{t_2-t_1}\int_{t_1}^{t_2} u(t)\mathrm{d}t$ $\quad m_u(n_1,n_2) = \frac{1}{n_2-n_1+1}\sum_{\nu=n_1}^{n_2} u(\nu)$ $\quad m_u = m_u(-\infty,\infty) = m_{u_T}(\tau,\tau+T)$

Gleichrichtmittelwert

$m_{|u|}(t_1,t_2) = \frac{1}{t_2-t_1}\int_{t_1}^{t_2} |u(t)|\,\mathrm{d}t$ $\quad m_{|u|}(n_1,n_2) = \frac{1}{n_2-n_1+1}\sum_{\nu=n_1}^{n_2} |u(\nu)|$ $\quad m_{|u|} = m_{|u|}(-\infty,\infty) = m_{|u_T|}(\tau,\tau+T)$

Effektivwert

$m_{u^2}(t_1,t_2) = \frac{1}{t_2-t_1}\int_{t_1}^{t_2} u(t)^2\mathrm{d}t$ $\quad m_{u^2}(n_1,n_2) = \frac{1}{n_2-n_1+1}\sum_{\nu=n_1}^{n_2} u(\nu)^2$ $\quad U_{\mathrm{eff}} = m_{u^2} = m_{u^2}(-\infty,\infty) = m_{u_T^2}(\tau,\tau+T)$

Energie

$W_u(t_1,t_2) = \frac{1}{Z}\int_{t_1}^{t_2} u(t)^2\mathrm{d}t = \frac{1}{Z}r_{uu}(t_1,t_2)(0)$ $\quad W_u(n_1,n_2) = \frac{1}{Z}\sum_{\nu=n_1}^{n_2} u(\nu)^2 = \frac{1}{Z}r_{uu}(n_1,n_2)(0)$ $\quad W_u = W_u(-\infty,\infty) = W_{u_T}(\tau,\tau+T) = \frac{1}{Z}r_{uu}(0)$

Leistung

$P_u(t_1,t_2) = \frac{1}{t_2-t_1}W_u(t_1,t_2)$ $\quad P_u(n_1,n_2) = \frac{1}{n_2-n_1+1}W_u(n_1,n_2)$ $\quad P_u = P_u(-\infty,\infty) = P_{u_T}(\tau,\tau+T)$

Varianz

$\sigma_u^2(t_1,t_2) = \frac{1}{t_2-t_1}\int_{t_1}^{t_2}(u(t)-m_u(t_1,t_2))^2\,\mathrm{d}t = ZP_u(t_1,t_2) - m_u(t_1,t_2)^2$

$\sigma_u^2(n_1,n_2) = \frac{1}{n_2-n_1+1}\sum_{\nu=n_1}^{n_2}(u(\nu)-m_u(n_1,n_2))^2 = ZP_u(n_1,n_2) - m_u(n_1,n_2)^2$

$\sigma_u^2 = \sigma_u^2(-\infty,\infty) = \sigma_{u_T}^2(\tau,\tau+T) = ZP_u - m_u^2$

Kreuzkorrelationsfunktion

$r_{uv}(t_1,t_2)(t) = \int_{t_1}^{t_2} u(\tau)v(t+\tau)\mathrm{d}\tau$ $\quad r_{uv}(n_1,n_2)(n) = \sum_{\nu=n_1}^{n_2-n} u(\nu)v(n+\nu)$ $\quad r_{uv}(t) = r_{uv}(-\infty,\infty)(t) = r_{u_T v_T}(\tau,\tau+T)(t)$

$\mathcal{F}(r_{uv}(t)) = U(\omega)\overline{V}(\omega)$

normiert

$\rho_{uv}(t) = \frac{r_{uv}(t)}{r_{uv}(0)}$

Leistungssignale

${}^L r_{uv}(t) = \lim_{T\to\infty}\left(\frac{1}{T}\int_T u(\tau)v(t+\tau)\mathrm{d}\tau\right)$

Signal-Verhältnis

$\mathrm{SR}(u,v) = \frac{P_u}{P_v} = \frac{U_{\mathrm{eff}}^2}{V_{\mathrm{eff}}^2}$

Signal-Abstand

$\mathrm{SR}_{\mathrm{dB}}(u,v) = \log_{10}\left(\frac{P_u}{P_v}\right)[\mathrm{B}] = 2\log_{10}\left(\frac{U_{\mathrm{eff}}}{V_{\mathrm{eff}}}\right)[\mathrm{B}]$

Hilberttransformation

$\mathcal{H}(u(t)) = u(t) \star \frac{1}{t} = \int_{-\infty}^{\infty}\frac{u(\tau)}{t-\tau}\mathrm{d}\tau$ $\quad \mathcal{H}(\mathcal{H}(u(t))) = u(t) \star \frac{1}{t} \star \frac{1}{t} = -\pi^2 u(t)$

Digitalisierung

Ideale Abtastung

$u^\star(t) = u/t)\delta_T(t) = \sum_{n=-\infty}^{\infty} u(nT)\delta(t-nT) \quad ○\!\!-\!\!\bullet \quad U^\star(\omega) = \frac{1}{2\pi}U(\omega) \star \omega_T\delta_{\omega_T}(\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} U(\omega - k\omega_T)$

$u(t) = u^\star(t) \star T h_{\text{TP}}\left(\frac{\omega_T}{2}\right)(t) = \sum_{n=-\infty}^{\infty} u(nT)\text{si}\left(\frac{\omega_T}{2}(t-nT)\right) \quad ○\!\!-\!\!\bullet \quad U(\omega) = U^\star(\omega) T H_{\text{TP}}\left(\frac{\omega_T}{2}\right)(\omega)$

Signalausblendung (shape-top sampling, natural sampling) mit $0 < \alpha \le 1$

$u_a(t) = u(t)\left(\Pi_{\alpha T}(t) \star \delta_T(t)\right) \quad ○\!\!-\!\!\bullet \quad U_a(\omega) = \frac{1}{2\pi}U(\omega) \star \alpha T \text{si}\left(\frac{\omega\alpha T}{2}\right)\omega_T\delta_{\omega_T}(\omega) = \alpha \sum_{k=-\infty}^{\infty} \text{si}(k\pi\alpha)U(\omega - k\omega_T)$

$u(t) = u_a(t) \star \frac{1}{\alpha}h_{\text{TP}}\left(\frac{\omega_T}{2}\right)(t) \quad ○\!\!-\!\!\bullet \quad U(\omega) = U_a(\omega)\frac{1}{\alpha}H_{\text{TP}}\left(\frac{\omega_T}{2}\right)(\omega)$

Signalverbreiterung (flat-top sampling)

$u_a(t) = u^\star(t) \star \Pi_{\alpha T}(t) \quad ○\!\!-\!\!\bullet \quad U_a(\omega) = \alpha \text{si}\left(\frac{\omega\alpha T}{2}\right)\sum_{k=-\infty}^{\infty} U(\omega - k\omega_T)$

Abtastrate	Bandpaßsignale
$f_T \ge 2f_g$	$f_T = \frac{2}{n}f_{g\max}$ mit $n \in \mathbb{N} \wedge \frac{f_{g\max}}{f_B} - 1 < n \le \frac{f_{g\max}}{f_B}$

Quantisierungsrauschen $q(t)$ (Quantisierungsstufenhöhe Δ, Bitanzahl m)

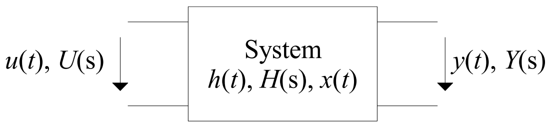
$P_q = \frac{1}{Z}\frac{\Delta^2}{12} = \frac{1}{Z}\frac{\hat{u}^2}{2^{2m}\cdot 3} \qquad \text{SNR}(u) = \text{SR}(u,q) = \alpha 2^{2m} \qquad \text{SNR}_{\text{dB}}(u) = 0{,}602m + \log_{10}(\alpha)\,[\text{B}]$

Spezielle Systeme

Bezeichnung	**Definition/Eigenschaften**	**bei Pol-/Nullstellen**	**bei diskreten PN**
Stabil	$W_h < \infty$	$\Re(s_\infty) < 0$	$\|z_\infty\| < 1$
Kausal	$h(t<0) = 0 \qquad \int_{-\infty}^{\infty} \frac{\|\ln(A(\omega))\|}{1+\frac{\omega^2}{\omega_B^2}}\mathrm{d}\omega < \infty$ $h(t) = h_g(t) + h_u(t) \quad ○\!\!-\!\!\bullet \quad H(\omega) = H_g(\omega) + H_u(\omega)$ $h_g(t) = h_u(t)\text{sgn}(t) \quad ○\!\!-\!\!\bullet \quad H_g(\omega) = \frac{1}{\mathrm{i}\pi}\mathcal{H}(H_u(\omega))$	$R \le Q$ (reale Systeme immer)	$R \le Q$
Linearphasig *(Stabil)*	$H_{\text{LP}}(\omega) = A(\omega)\exp(-\mathrm{i}\omega t_0) \qquad \varphi_{\text{LP}}(\omega) = -\omega t_0$ $h(t_0+t) = h(t_0-t) \quad \vee \quad h(t_0+t) = -h(t_0-t)$	$\Re(s_0) = -\Re(s_0) \quad Q=0$ (reale Systeme nie)	$z_0 = \overline{z_0^{-1}} \quad Q=0$ $t_0 = TR/2$
Allpaß *(Stabil)*	$H_{\text{AP}}(\omega) = A_0\exp(\mathrm{i}\varphi(\omega)) \qquad A_{\text{AP}}(\omega) = A_0$	$\Re(s_0) = -\Re(s_\infty) \quad R=Q$ $A_0 = \left\|\frac{b_R}{a_Q}\right\| = \left\|\frac{b_0}{a_0}\right\|$	$z_0 = \overline{z_\infty^{-1}} \quad R=Q$ $A_0 = \left\|\frac{b_0}{a_0}\right\|\prod_{r=1}^{R}\|z_{0r}\|$ $= \left\|\frac{b_0}{a_0}\right\|\prod_{q=1}^{Q}\left\|\overline{z_{\infty q}^{-1}}\right\|$ $b_r = H_{\text{AP}}\,a_{R-r}$
Verzerrungsfrei *(Linearphasig und Allpaß)*	$y_{\text{VS}}(t) = \alpha u(t-t_0) \qquad H_{\text{VS}}(\omega) = A_0\exp(-\mathrm{i}\omega t_0)$		
Minimalphasig/ Allpaßfrei *(Kausal)*	$H_{\min}(\omega) = A(\omega)\exp(\mathrm{i}\varphi_{\min}(\omega))$ $\varphi_{\min}(\omega) = \mathcal{H}(\ln(A(\omega)))$	$\Re(s_{0\|\infty}) < 0$	$\left\|z_{0\|\infty}\right\| < 1$

Nichtrekursiv	$Q = 0$	$h(n) = \frac{1}{a_0}\sum_{r=0}^{R} b_r\delta(n-r) \quad ○\!\!-\!\!\bullet \quad H(z) = \frac{1}{a_0}\sum_{r=0}^{R} b_r z^{-r} = \frac{b_0}{a_0}\frac{\prod_{r=1}^{R}(z-z_{0r})}{z^R}$
Reinrekursiv	$R = 0$	$h(n) \quad ○\!\!-\!\!\bullet \quad H(z) = \frac{b_0}{\sum_{q=0}^{Q} a_q z^{-q}} = \frac{b_0}{a_0}\frac{z^Q}{\prod_{q=1}^{Q}(z-z_{\infty q})}$

Systembeschreibung (Vierpol)



Vierpolparameter

H-Parameter
$$\begin{pmatrix} u_e \\ i_a \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \cdot \begin{pmatrix} i_e \\ u_a \end{pmatrix} \qquad |\mathbf{H}| = \frac{1}{y_{11}} \begin{pmatrix} 1 & -y_{12} \\ y_{21} & |\mathbf{Y}| \end{pmatrix} = \frac{1}{z_{22}} \begin{pmatrix} |\mathbf{Z}| & z_{12} \\ -z_{21} & 1 \end{pmatrix}$$

Z-Parameter
$$\begin{pmatrix} u_e \\ u_a \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} \cdot \begin{pmatrix} i_e \\ i_a \end{pmatrix} \qquad |\mathbf{Z}| = \frac{1}{h_{22}} \begin{pmatrix} |\mathbf{H}| & h_{12} \\ -h_{21} & 1 \end{pmatrix} = \frac{1}{|\mathbf{Y}|} \begin{pmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{pmatrix}$$

Y-Parameter
$$\begin{pmatrix} i_e \\ i_a \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \cdot \begin{pmatrix} u_e \\ u_a \end{pmatrix} \qquad |\mathbf{Y}| = \frac{1}{h_{11}} \begin{pmatrix} 1 & -h_{12} \\ h_{21} & |\mathbf{H}| \end{pmatrix} = \frac{1}{|\mathbf{Z}|} \begin{pmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{pmatrix}$$

LTI-System (Linear Time Invariant)

Frequenzgang

$$H(\omega) = \frac{Y(\omega)}{U(\omega)} = A_h(\omega)\exp\left(\mathrm{i}\varphi_h(\omega)\right) = H(\mathrm{s})|_{\mathrm{s}=\mathrm{i}\omega} = H(\mathrm{z})|_{\mathrm{z}=\exp(\mathrm{i}\omega T)}$$

Amplitudengang

$A_h(\omega) = |H(\omega)|$

Dämpfung=−Verstärkung

$a_h(\omega) = \mathrm{SR}_{\mathrm{dB}}(U, Y) = -2\log_{10}\left(|H(\omega)|\right)[\mathrm{B}] = -v_h(\omega)$

Phasengang

$\varphi_h(\omega) = \arg\left(H(\omega)\right) = -\omega t_{\mathrm{phase}}(\omega)$

Phasenlaufzeit

$t_{\mathrm{phase}}(\omega) = -\frac{\varphi_h(\omega)}{\omega}$

Systemantworten

$u(t) \rightarrow y(t) = u(t) \star h(t) \quad ○\!\!-\!\!\bullet \quad Y(\mathrm{s}) = U(\mathrm{s})H(\mathrm{s}) \qquad u(n) \rightarrow y(n) = u(n) \star h(n) \quad ○\!\!-\!\!\bullet \quad Y(\mathrm{z}) = U(\mathrm{z})H(\mathrm{z})$

Impulsantwort

$\delta(t) \rightarrow h(t) = \frac{\mathrm{d}h_\sigma(t)}{\mathrm{d}t} \quad ○\!\!-\!\!\bullet \quad H(\mathrm{s}) = \mathrm{s}H_\sigma(\mathrm{s}) \qquad \delta(n) \rightarrow h(n) = h_\sigma(n) - h_\sigma(n-1) \quad ○\!\!-\!\!\bullet \quad H(\mathrm{z}) = \left(1 - \mathrm{z}^{-1}\right)H_\sigma(\mathrm{z})$

Sprungantwort

$\sigma(t) \rightarrow h_\sigma(t) = \int_{-\infty}^{t} h(\tau)\mathrm{d}\tau \quad ○\!\!-\!\!\bullet \quad H_\sigma(\mathrm{s}) = \mathrm{s}^{-1}H(\mathrm{s}) \qquad \sigma(n) \rightarrow h_\sigma(n) = \sum_{\nu=-\infty}^{n} h(n) \quad ○\!\!-\!\!\bullet \quad H_\sigma(\mathrm{z}) = \left(1 - \mathrm{z}^{-1}\right)^{-1} H(\mathrm{z})$

Exponentialsignale (unendlich ausgedehnt bzw. im eingeschwungenen Zustand)

$u(t) = u_0\exp(\mathrm{i}\omega_1 t) \rightarrow y(t) = u(t)H(\omega_1)$

$\Rightarrow H(\omega) = \frac{y(t)}{u(t)} = \frac{Z_y}{Z_u}$

$u(n) = u_0\exp(\mathrm{i}\omega_1 nT) \rightarrow y(n) = u(n)H(\omega_1)$

$\Rightarrow H(\omega) = \frac{y(n)}{u(n)}$

Pol-Nullstellen-Darstellung (Laplace-/Z-Transformation)

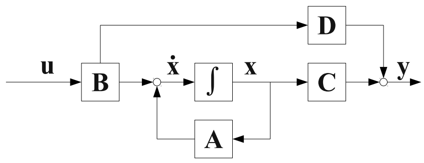
DGL

$$\sum_{q=0}^{Q} a_q \frac{\mathrm{d}^q y(t)}{\mathrm{d}t^q} = \sum_{r=0}^{R} b_r \frac{\mathrm{d}^r u(t)}{\mathrm{d}t^r} \qquad \sum_{q=0}^{Q} a_q y(n-q) = \sum_{r=0}^{R} b_r u(n-r)$$

Übertragungsfunktion

$$H(\mathrm{s}) = \frac{Y(\mathrm{s})}{U(\mathrm{s})} = \frac{Z(\mathrm{s}) = \sum_{r=0}^{R} b_r \mathrm{s}^r}{N(\mathrm{s}) = \sum_{q=0}^{Q} a_q \mathrm{s}^q} = \frac{b_R}{a_Q} \frac{\prod_{r=1}^{R}(\mathrm{s} - \mathrm{s}_{0r})}{\prod_{q=1}^{Q}(\mathrm{s} - \mathrm{s}_{\infty r})} \qquad H(\mathrm{z}) = \frac{Y(\mathrm{z})}{U(\mathrm{z})} = \frac{\sum_{r=0}^{R} b_r \mathrm{z}^{-r}}{\sum_{q=0}^{Q} a_q \mathrm{z}^{-q}} = \mathrm{z}^{Q-R}\frac{b_0}{a_0}\frac{\prod_{r=1}^{R}(\mathrm{z} - \mathrm{z}_{0r})}{\prod_{q=1}^{Q}(\mathrm{z} - \mathrm{z}_{\infty r})}$$

Zustandsmodell



$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t)$ **x** Zustandsvektor **u** Eingangsgrößen

$\mathbf{y}(t) = \mathbf{g}(\mathbf{x}(t), \mathbf{u}(t), t)$ **y** Ausgangsgrößen

Linearisierung im Arbeitspunkt AP

$$\dot{\mathbf{x}} \approx \mathbf{f}(\mathbf{x}_{\mathrm{AP}}, \mathbf{u}_{\mathrm{AP}}) + \mathbf{A}\cdot(\mathbf{x}-\mathbf{x}_{\mathrm{AP}}) + \mathbf{B}\cdot(\mathbf{u}-\mathbf{u}_{\mathrm{AP}}) \qquad \mathbf{A} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}^{\mathrm{T}}}(\mathbf{x}_{\mathrm{AP}}, \mathbf{u}_{\mathrm{AP}}) \qquad \mathbf{B} = \frac{\partial \mathbf{f}}{\partial \mathbf{u}^{\mathrm{T}}}(\mathbf{x}_{\mathrm{AP}}, \mathbf{u}_{\mathrm{AP}})$$

$$\mathbf{y} \approx \mathbf{g}(\mathbf{x}_{\mathrm{AP}}, \mathbf{u}_{\mathrm{AP}}) + \mathbf{C}\cdot(\mathbf{x}-\mathbf{x}_{\mathrm{AP}}) + \mathbf{D}\cdot(\mathbf{u}-\mathbf{u}_{\mathrm{AP}}) \qquad \mathbf{C} = \frac{\partial \mathbf{g}}{\partial \mathbf{x}^{\mathrm{T}}}(\mathbf{x}_{\mathrm{AP}}, \mathbf{u}_{\mathrm{AP}}) \qquad \mathbf{D} = \frac{\partial \mathbf{g}}{\partial \mathbf{u}^{\mathrm{T}}}(\mathbf{x}_{\mathrm{AP}}, \mathbf{u}_{\mathrm{AP}})$$

$\mathbf{h}(t) = \mathbf{C}\cdot\mathbf{\Psi}(t)\cdot\mathbf{B} + \mathbf{D}\delta(t) \quad \circ\!\!-\!\!\bullet \quad \mathbf{H}(s) = \mathbf{C}\cdot(s\mathbf{1}-\mathbf{A})^{-1}\cdot\mathbf{B}+\mathbf{D}$ Impulsantwort

$\mathbf{h}_\sigma(t) = \int_0^t \mathbf{C}\cdot\mathbf{\Psi}(\tau)\cdot\mathbf{B}\,\mathrm{d}\tau + \mathbf{D} = \mathbf{C}\cdot\mathbf{A}^{-1}\mathbf{\Psi}(t)\cdot\mathbf{B} - \mathbf{C}\cdot\mathbf{A}^{-1}\cdot\mathbf{B} + \mathbf{D}\sigma(t) \quad \circ\!\!-\!\!\bullet \quad \mathbf{H}_\sigma(s) = s^{-1}\mathbf{C}\cdot(s\mathbf{1}-\mathbf{A})^{-1}\cdot\mathbf{B}+\mathbf{D}$ Sprungantwort

$\mathbf{x}(t) = \mathbf{\Psi}(t)\cdot\mathbf{x}(0) + \int_0^t \mathbf{\Psi}(t-\tau)\cdot\mathbf{B}\cdot\mathbf{u}(\tau)\,\mathrm{d}\tau \qquad \mathbf{\Psi}(t) = \exp(\mathbf{A}t)$ Bewegungsgleichung

$\mathbf{y}(t) = \mathbf{C}\cdot\mathbf{\Psi}(t)\cdot\mathbf{x}(0) + \int_0^t \mathbf{C}\cdot\mathbf{\Psi}(t-\tau)\cdot\mathbf{B}\cdot\mathbf{u}(\tau)\,\mathrm{d}\tau + \mathbf{D}\cdot\mathbf{u}(t)$ Ausgangsgleichung

$\mathbf{K}_s = -\mathbf{C}\cdot\mathbf{A}^{-1}\cdot\mathbf{B}+\mathbf{D}$ statische Verstärkung

Regelungsnormalform ($R \le Q$)

$$\mathbf{x}=\frac{a_q}{b_0}\begin{pmatrix} y \\ \vdots \\ \frac{\mathrm{d}^{Q-1}y}{\mathrm{d}t^{Q-1}}\end{pmatrix} \quad \mathbf{A}=\begin{pmatrix} 0 & 1 & & 0 \\ \vdots & & \ddots & \\ 0 & 0 & & 1 \\ -\frac{a_0}{a_Q} & -\frac{a_1}{a_Q} & \cdots & -\frac{a_{Q-1}}{a_Q}\end{pmatrix} \quad \mathbf{B}=\begin{pmatrix}0\\ \vdots \\ 0 \\ 1\end{pmatrix} \quad \mathbf{C}=\begin{pmatrix}\frac{b_0}{a_Q}-\frac{b_Q a_0}{a_Q^2} & \cdots & \frac{b_{Q-1}}{a_Q}-\frac{b_Q a_{Q-1}}{a_Q^2}\end{pmatrix} \quad \mathbf{D} = \frac{b_Q}{a_Q} \quad K_s = \frac{b_0}{a_0}$$

Beobachtungsnormalform ($R \le Q$)

$$\mathbf{A}=\begin{pmatrix} 0 & \cdots & 0 & -a_0/a_Q \\ 1 & & 0 & -a_1/a_Q \\ & \ddots & & \vdots \\ 0 & & 1 & -a_{Q-1}/a_Q\end{pmatrix} \quad \mathbf{B}=\begin{pmatrix} b_0/a_Q - b_n a_0/a_Q^2 \\ \vdots \\ b_{Q-1}/a_Q - b_n a_{Q-1}/a_Q^2\end{pmatrix} \quad \mathbf{C}=\begin{pmatrix}0 & \cdots & 0 & 1\end{pmatrix} \quad \mathbf{D} = \frac{b_Q}{a_Q} \quad K_s = \frac{b_0}{a_0}$$

Kanonische Normalform (λ_q Eigenwerte, $\boldsymbol{\lambda}_q$ Eigenvektoren von **A**)

$$\mathbf{x}' = \mathbf{V}^{-1}\cdot\mathbf{x} \quad \mathbf{A}'=\mathbf{V}^{-1}\cdot\mathbf{A}\cdot\mathbf{V} = \mathrm{diag}(\lambda_q) \quad \mathbf{B}' = \mathbf{V}^{-1}\cdot\mathbf{B} \quad \mathbf{C}' = \mathbf{C}\cdot\mathbf{V} \quad \mathbf{D}' = \mathbf{D} \quad \mathbf{V}=\begin{pmatrix}\boldsymbol{\lambda}_1 & \cdots & \boldsymbol{\lambda}_Q\end{pmatrix}$$

Eigenbewegung Eigenlösung

$$\mathbf{\Psi}'(t) = \mathrm{diag}\left(\exp(\lambda_q t)\right) \quad \mathbf{\Psi}'(t)\cdot\mathbf{x}'(0) = \begin{pmatrix}\exp(\lambda_1 t)x_1' \\ \vdots \\ \exp(\lambda_q t)x_q'\end{pmatrix} \quad \mathbf{\Psi}(t)\cdot\mathbf{x}(0) = \sum_{q=1}^{Q}\boldsymbol{\lambda}_q \exp(\lambda_q t)x_q'(0) \quad \mathbf{x}_q(t) = \boldsymbol{\lambda}_q\exp(\lambda_q t)$$

Stabilität

Ljapunow

$\mathrm{stabil}_{\mathrm{Ljapunow}} \Leftrightarrow \forall\varepsilon > 0 : \exists\delta > 0 : \forall\,\|\mathbf{x}(0)\| < \delta : \forall t > 0 : \|\mathbf{x}(t)\| < \varepsilon \;\Leftarrow\; \forall\lambda_q : \Re(\lambda_q) \le 0$

$\text{asymptotisch stabil}_{\mathrm{Ljapunow}} \Leftrightarrow \mathrm{stabil}_{\mathrm{Ljapunow}} \wedge \lim_{t\to\infty}(\|\mathbf{x}(t)\|) = 0 \;\Leftarrow\; \forall\lambda_q : \Re(\lambda_q) < 0$

Hurwitz (asymptotisch stabil)

$$\det(\mathbf{A}-\lambda\mathbf{1}) = \sum_{i=0}^{n} a_i\lambda^i = 0 \qquad D_0 = a_1 \qquad D_1 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2\end{vmatrix} \qquad D_2 = \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3\end{vmatrix} \qquad \cdots$$

$\mathrm{stabil}_{\mathrm{Hurwitz}} \Leftrightarrow (\forall a_i|0 \le i \le n : a_i > 0) \wedge (\forall D_i|0 \le i \le n : D_i > 0)$

Routh (asymptotisch stabil)

$\text{zeile } 1 \mathrel{\hat{=}} a_{n-0} \quad a_{n-2} \quad a_{n-4} \quad \cdots \quad 0 \qquad \text{zeile } j \mathrel{\hat{=}} z_{j,1} \quad z_{j,2} \quad \cdots$

$\text{zeile } 2 \mathrel{\hat{=}} a_{n-1} \quad a_{n-3} \quad a_{n-5} \quad \cdots \quad 0 \qquad z_{j,1} = z_{j-2,i+1} - t_j z_{j-1,i+1} \qquad t_j = \frac{z_{j-2,1}}{z_{j-1,1}}$

$\mathrm{stabil}_{\mathrm{Routh}} \Leftrightarrow (\forall a_i|0 \le i \le n : a_i > 0) \wedge \left(\forall z_{j,1} : z_{j,1} > 0\right)$

BIBO (Bounded Input Bounded Output)

$\int_0^\infty |h(t)|\,\mathrm{d}t < \infty \;\Leftrightarrow\; \forall s_{\infty q} : \Re(s_{\infty q}) < 0 \;\Leftarrow\;$ asymptotisch stabil

Betrachtung der Schleifenverstärkung H_0 bei Einheitsrückführung ($Q > R$)

Nyquistkriterium

$\forall\omega : H_0(\mathrm{i}\omega) \ne \pm 1 \;\wedge\; \arg(1 \mp H_0(\mathrm{i}\infty)) - \arg(1 \mp H_0(\mathrm{i}0)) = n_r\pi + n_a\frac{\pi}{2} \qquad n_r = \#\left(s_{\infty q}|\Re(s_{\infty q}) > 0\right) \qquad n_a = \#\left(s_{\infty q}|\Re(s_{\infty q}) = 0\right)$

Phasenrand

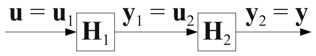
$\Phi_R > 0 \qquad \Phi_R = \left(\frac{3}{2} \pm \frac{1}{2}\right)\pi + \arg(H_0(\mathrm{i}\omega_c)) \qquad |H_0(\mathrm{i}\omega_c)| = 1$

Amplitudenrand

$k_R > 1 \qquad k_R = |H_0(\mathrm{i}\omega_{-2\pi})|^{-1} \qquad \arg(H_0(\mathrm{i}\omega_{-2\pi})) - \left(\frac{1}{2} \mp \frac{1}{2}\right)\pi = -2\pi$

Gekoppelte Systeme

Reihenschaltung



$A_h = A_{h_1} A_{h_2}$ $\qquad \phi_h = \phi_{h_1} + \phi_{h_2}$

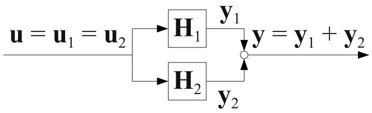
$a_h = a_{h_1} + a_{h_2}$ $\qquad v_h = v_{h_1} + v_{h_2}$

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_1 & \mathbf{0} \\ \mathbf{B}_2 \cdot \mathbf{C}_1 & \mathbf{A}_2 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \cdot \mathbf{D}_1 \end{pmatrix} \quad \mathbf{x} = \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} \mathbf{D}_2 \cdot \mathbf{C}_1 & \mathbf{C}_2 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} \mathbf{D}_2 \cdot \mathbf{D}_1 \end{pmatrix}$$

$\mathbf{h} = \mathbf{h}_2 \star \mathbf{h}_1$ $\qquad \mathbf{H} = \mathbf{H}_2 \cdot \mathbf{H}_1$

Parallelschaltung

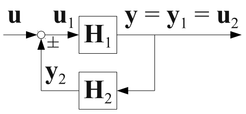


$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{pmatrix} \quad \mathbf{x} = \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} \mathbf{C}_1 & \mathbf{C}_2 \end{pmatrix} \quad \mathbf{D} = \mathbf{D}_1 + \mathbf{D}_2$$

$\mathbf{h} = \mathbf{h}_1 + \mathbf{h}_2$ $\qquad \mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2$

Rückkopplung



$\mathbf{H}_0 = \mathbf{H}_2 \cdot \mathbf{H}_1$ $\qquad$ Schleifenübertragungsfunktion

$\mathbf{H} = (\mathbf{1} \mp \mathbf{H}_1 \cdot \mathbf{H}_2)^{-1} \cdot \mathbf{H}_1$

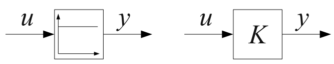
$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_1 \pm \mathbf{B}_1 \cdot \mathbf{D}_2 \cdot (\mathbf{1} \mp \mathbf{D}_1 \cdot \mathbf{D}_2)^{-1} \cdot \mathbf{C}_1 & \pm \mathbf{B}_1 \cdot \mathbf{C}_2 + \mathbf{B}_1 \cdot \mathbf{D}_2 \cdot (\mathbf{1} \mp \mathbf{D}_1 \cdot \mathbf{D}_2)^{-1} \cdot \mathbf{D}_1 \cdot \mathbf{C}_2 \\ \mathbf{B}_2 \cdot (\mathbf{1} \mp \mathbf{D}_1 \cdot \mathbf{D}_2)^{-1} \cdot \mathbf{C}_1 & \mathbf{A}_2 \pm \mathbf{B}_2 \cdot (\mathbf{1} \mp \mathbf{D}_1 \cdot \mathbf{D}_2)^{-1} \cdot \mathbf{D}_1 \cdot \mathbf{C}_2 \end{pmatrix} \quad \mathbf{x} = \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} \mathbf{B}_1 \pm \mathbf{B}_1 \cdot \mathbf{D}_2 \cdot (\mathbf{1} \mp \mathbf{D}_1 \cdot \mathbf{D}_2)^{-1} \cdot \mathbf{D}_1 \\ \mathbf{B}_2 \cdot (\mathbf{1} \mp \mathbf{D}_1 \cdot \mathbf{D}_2)^{-1} \cdot \mathbf{D}_1 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} (\mathbf{1} \mp \mathbf{D}_1 \cdot \mathbf{D}_2)^{-1} \cdot \mathbf{C}_1 & \pm (\mathbf{1} \mp \mathbf{D}_1 \cdot \mathbf{D}_2)^{-1} \cdot \mathbf{D}_1 \cdot \mathbf{C}_2 \end{pmatrix} \quad \mathbf{D} = (\mathbf{1} \mp \mathbf{D}_1 \cdot \mathbf{D}_2)^{-1} \cdot \mathbf{D}_1$$

Bausteine

P Proportionalblock

$y = K_s u$ $\qquad h_\sigma(t) = K_s \sigma(t)$ $\qquad H(\mathrm{s}) = K_s$

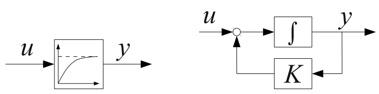


PT$_\mathbf{n}$ proportional im eingeschwungenen Zustand, n Zeitkonstanten

$\sum_{q=1}^{Q} T_q \frac{\mathrm{d}^q y}{\mathrm{d}t^q} + y = K_s u$ $\qquad H(\mathrm{s}) = \frac{K_s}{\sum_{q=1}^{Q} T_q \mathrm{s}^q + 1}$

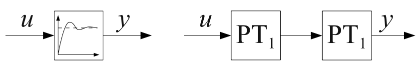
PT$_\mathbf{1}$ T = Zeitkonstante

$T\dot{y} + y = K_s u$ $\qquad h_\sigma(t) = K_s \left(1 - \exp\left(-\frac{t}{T}\right)\right) \sigma(t)$



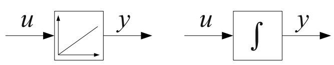
PT$_\mathbf{2}$ d = Dämpfung

$T^2 \ddot{y} + 2dT\dot{y} + y = K_s u$



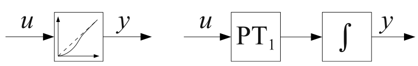
I Integralblock

$T_I \dot{y} = u$ $\qquad h_\sigma(t) = \frac{t}{T_I} \sigma(t)$ $\qquad H(\mathrm{s}) = \frac{1}{T_I \mathrm{s}}$



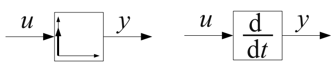
IT$_\mathbf{1}$ (PT$_1$ + I)

$T_I T \ddot{y} + T_I \dot{y} = u$ $\qquad h_\sigma(t) = \left(\frac{t}{T_I} - \frac{T}{T_I}\left(1 - \exp\left(-\frac{t}{T}\right)\right)\right) \sigma(t)$



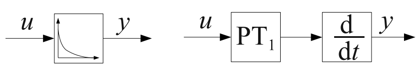
D Differentialblock

$y = T_D \dot{u}$ $\qquad h_\sigma(t) = T_D \delta(t)$ $\qquad H(\mathrm{s}) = \mathrm{s} T_D$



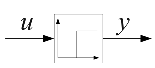
DT$_\mathbf{1}$ (PT$_1$ + D)

$T\dot{y} + y = T_D \dot{u}$



T$_\mathbf{t}$ Totzeitblock

$y(t) = K_s u(t - T_t)$ $\qquad h_\sigma(t) = K_s \sigma(t - T_t)$ $\qquad H(\mathrm{s}) = K_s \exp(-\mathrm{s} T_t)$



Filter

3 dB-Grenzfrequenz: $A_h(\omega_g) = \frac{1}{\sqrt{2}} A_h(\omega_0)$

äquivalente Bandbreite: $A_h(\omega_0)\omega_{\text{eq}} = \int\limits_{-\infty}^{\infty} A_h(\omega)\mathrm{d}\omega$

Transitfrequenz: $A_h(\omega_t) = 1$

Tiefpaß

$h_{\text{TP}}(\omega_g)(t) = \frac{\omega_g}{\pi}\,\text{si}\left(\omega_g(t-t_0)\right)$ $\qquad h_{\text{TP}}(\omega_g)(n) = \frac{\omega_g}{\pi}\,\text{si}\left(\omega_g(n-n_0)\right)$

$H_{\text{TP}}(\omega_g)(\omega) = \Pi_{2\omega_g}(\omega)\exp(-\mathrm{i}\omega t_0)$ $\qquad H_{\text{TP}}(\omega_g)(\omega) = \Pi_{2\omega_g}(\omega)\exp(-\mathrm{i}\omega n_0 T)$

Bandpaß (Mittenfrequenz $\omega_m = \left(\omega_{g\,\text{min}} + \omega_{g\,\text{max}}\right)/2$, Bandbreite $\omega_B = \omega_{g\,\text{max}} - \omega_{g\,\text{min}}$)

$h_{\text{BP}}(\omega_B)(t) = 2h_{\text{TP}}\left(\frac{\omega_B}{2}\right)(t)\cos(\omega_m t)$ $\qquad h_{\text{BP}}(\omega_B)(n) = 2h_{\text{TP}}\left(\frac{\omega_B}{2}\right)(n)\cos(n\omega_m T)$

$H_{\text{BP}}(\omega_B)(\omega) = H_{\text{TP}}\left(\frac{\omega_B}{2}\right)(\omega-\omega_m) + H_{\text{TP}}\left(\frac{\omega_B}{2}\right)(\omega+\omega_m)$ $\qquad H_{\text{BP}}(\omega_B)(\omega) = H_{\text{TP}}\left(\frac{\omega_B}{2}\right)(\omega-\omega_m) + H_{\text{TP}}\left(\frac{\omega_B}{2}\right)(\omega+\omega_m)$

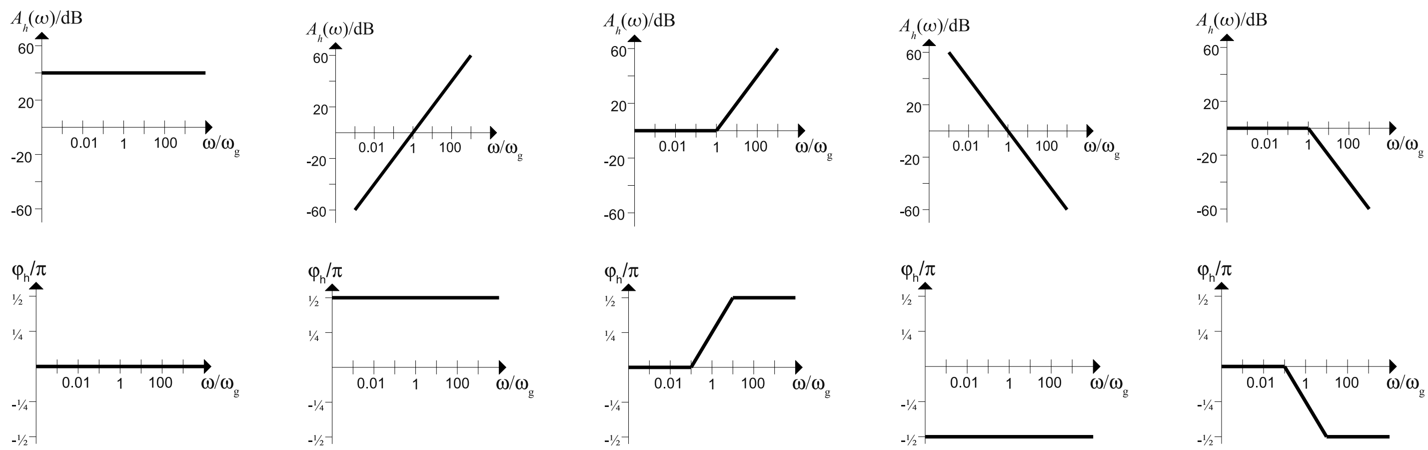
Hochpaß

$h_{\text{HP}}(\omega_g)(t) = \delta(t-t_0) - \frac{\omega_g}{\pi}\,\text{si}\left(\omega_g(t-t_0)\right)$ $\qquad h_{\text{HP}}(\omega_B)(n) = h_{\text{TP}}(\omega_B)(n)\cos(n\pi)$

$H_{\text{HP}}(\omega_g)(\omega) = \left(1-\Pi_{2\omega_g}(\omega)\right)\exp(-\mathrm{i}\omega t_0)$ $\qquad H_{\text{HP}}(\omega_B)(\omega) = H_{\text{TP}}(\omega_B)\left(\omega \pm \frac{\pi}{T}\right)$

Bode-Diagramm

$H = \text{const}$ $\qquad H = \mathrm{i}\omega/\omega_0$ $\qquad H = 1 + \mathrm{i}\omega/\omega_0$ $\qquad H = \frac{1}{\mathrm{i}\omega/\omega_0}$ $\qquad H = \frac{1}{1+\mathrm{i}\omega/\omega_0}$



Allgemeines Filter

$H(\mathrm{s}) = \frac{H_0}{1+\sum\limits_{n=1}^{N} c_n \mathrm{S}'^n}$ $\quad c_n \in \mathbb{R}_0^+$

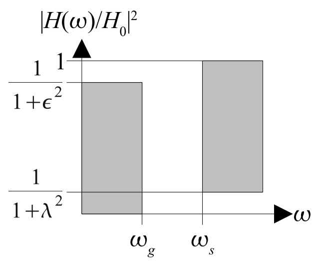
Tiefpaß: $\mathrm{S}' = \mathrm{S}$

Hochpaß: $\mathrm{S}' = \frac{1}{\mathrm{S}}$ $\qquad \mathrm{S} = \frac{\mathrm{s}}{\omega_g}$

Bandpaß: $\mathrm{S}' = \frac{1}{\Delta\Omega}\left(\mathrm{S} + \frac{1}{\mathrm{S}}\right)$

Bandsperre: $\mathrm{S}' = \frac{\Delta\Omega}{\mathrm{S}+\frac{1}{\mathrm{S}}}$ $\qquad \mathrm{S} = \frac{\mathrm{s}}{\omega_0}$ $\qquad \Delta\Omega = \frac{\omega_{g_2}-\omega_{g_1}}{\omega_0}$

Filterentwurf



Butterworth-Approximation

$|H(\omega)| = \frac{|H_0|}{\sqrt{1+\epsilon^2\Omega^{2N}}}$ $\qquad \mathrm{s}_{\infty n} = \frac{\omega_g}{\sqrt[N]{\epsilon}}\exp\left(\mathrm{i}\frac{\pi}{2}\left(1+\frac{2n-1}{N}\right)\right)$ $\qquad N \geq \frac{\log\left(\frac{\lambda}{\epsilon}\right)}{\log\left(\frac{\omega_s}{\omega_g}\right)}$

Tschebyscheff-Approximation

$|H(\omega)| = \frac{|H_0|}{\sqrt{1+\epsilon^2 T_N(\Omega)^2}}$ $\qquad T_N(x) = 2xT_{n-1}(x) - T_{n-2}(x)$, $\quad T_0(x) = 1 \quad T_1(x) = x$ $\qquad N \geq \frac{\operatorname{arccos}\left(\frac{\lambda}{\epsilon}\right)}{\operatorname{arccos}\left(\frac{\omega_s}{\omega_g}\right)}$

Cauer-Approximation

$|H(\omega)| = \frac{|H_0|}{\sqrt{1+\epsilon^2 \Psi_N(\Omega)^2}}$ $\qquad \Psi(x)$ elliptische Jacobi-Funktion

Bessel-Approximation

$H(\mathrm{s}) = H_0 \frac{B_N(0)}{B_N(\mathrm{s}t_{\text{phase}})}$ $\qquad B_N(x) = (2n-1)\,B_{n-1}(x) + B_{n-2}(x)$, $\quad B_0(x) = 1 \quad B_1(x) = x + 1$