Elektrisches Feld

$\nabla \times \mathbf{E} = -\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}$ $\qquad$ $\oint \mathbf{E} \cdot \mathrm{d}\mathbf{s} = -\frac{\mathrm{d}}{\mathrm{d}t} \int \mathbf{B} \cdot \mathrm{d}\mathbf{A}$

$\nabla \cdot \mathbf{D} = \rho_V$ $\qquad$ $\oint \mathbf{D} \cdot \mathrm{d}\mathbf{A} = \int \rho_V \mathrm{d}V$

$\mathbf{F} = q\mathbf{E} = (\mathbf{p}_e \cdot \nabla)\,\mathbf{E}$ $\qquad$ $\mathbf{T} = \mathbf{p}_e \times \mathbf{E}$

$W = \frac{1}{2}CU^2 = \frac{1}{2}QU = -\mathbf{p}_e \cdot \mathbf{E}$ $\qquad$ $w_V = \frac{1}{2}\mathbf{D} \cdot \mathbf{E} = \frac{1}{2}\rho_V \phi$

$P = UI$

$\mathbf{D} = \frac{\mathrm{d}Q}{\mathrm{d}\mathbf{A}} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon \mathbf{E}$ Verschiebungsdichte/Erregung

$\mathbf{E} = \frac{\mathrm{d}U}{\mathrm{d}\mathbf{s}} = \gamma^{-1}\mathbf{J} = \varepsilon^{-1}\mathbf{D} = \varepsilon_0^{-1}(\mathbf{D} - \mathbf{P}) = -\nabla\phi - \frac{\partial \mathcal{A}}{\partial t} = \frac{\mathrm{d}\mathbf{F}}{\mathrm{d}q}$ Feldstärke

$\mathbf{P} = n\mathbf{p}_e = \mathbf{D} - \varepsilon_0 \mathbf{E} = \varepsilon_0 \chi_e \mathbf{E}$ Polarisation

$\phi = \int_{\mathbf{r}_0}^{\mathbf{r}} \mathbf{E} \cdot \mathrm{d}\mathbf{s} = 2\frac{w_V}{\rho_V}$ Potential

$\varepsilon = \varepsilon_r \varepsilon_0 = (1 + \chi_e)\,\varepsilon_0 = \frac{\mathbf{D}}{\mathbf{E}}$ Dielektrizität

$\gamma = \frac{1}{\rho} = \frac{\mathbf{J}}{\mathbf{E}} = nq\mu$ spezifischer Leitwert

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$\rho_V = \frac{\mathrm{d}Q}{\mathrm{d}V} = \frac{\mathbf{J}}{\mathbf{v}}$ $\qquad$ $\rho_{V_{\mathrm{pol}}} = -\nabla \cdot \mathbf{P}$

$\rho_A = \frac{\mathrm{d}Q}{\mathrm{d}A}$ $\qquad$ $\rho_{A_{\mathrm{pol}}} = \frac{\mathbf{P} \cdot \mathbf{A}}{|\mathbf{A}|}$

$\mathbf{p}_e = Q\mathbf{d}$

$U = \phi_2 - \phi_1 = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E} \cdot \mathrm{d}\mathbf{s} = -\frac{\mathrm{d}\psi}{\mathrm{d}t} = \pm L\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{I}{Y}$

$I = \int \mathbf{J} \cdot \mathrm{d}\mathbf{A} = \frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{V}{N} = C\frac{\mathrm{d}U}{\mathrm{d}t} = UY$

$Q = \oint \mathbf{D} \cdot \mathrm{d}\mathbf{A} = \oint I \mathrm{d}t = CU$ Gaußscher Satz

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$C = \frac{Q}{U} = \varepsilon\frac{A}{d} = 2\frac{W}{U^2}$ Kapazität

$Y = \frac{1}{R} = \frac{I}{U} = \gamma\frac{A}{d} = \mathrm{i}\omega C = \frac{1}{\mathrm{i}\omega L}$ Admittanz (Leitwert)

Magnetisches Feld

$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\mathrm{d}\mathbf{D}}{\mathrm{d}t}$ $\qquad$ $\oint \mathbf{H} \cdot \mathrm{d}\mathbf{s} = \int \left(\mathbf{J} + \frac{\mathrm{d}\mathbf{D}}{\mathrm{d}t}\right) \cdot \mathrm{d}\mathbf{A}$

$\nabla \cdot \mathbf{B} = 0$ $\qquad$ $\oint \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$

$\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B}) = (\mathbf{p}_m \cdot \nabla)\,\mathbf{B}$ $\qquad$ $\mathbf{T} = \mathbf{p}_m \times \mathbf{B}$

$W = \frac{1}{2}LI^2 = \frac{1}{2}\phi V = -\mathbf{p}_m \cdot \mathbf{B}$ $\qquad$ $w_V = \frac{1}{2}\mathbf{B} \cdot \mathbf{H} = \frac{1}{2}\mathbf{J} \cdot \mathcal{A}$

$p_V = \mathbf{J} \cdot \mathbf{E}$

$\mathbf{B} = \frac{\mathrm{d}\phi}{\mathrm{d}\mathbf{A}} = \mu_0\,(\mathbf{H} + \mathbf{M}) = \mu\mathbf{H} = \nabla \times \mathcal{A} = \frac{\mathbf{F}}{q|\mathbf{v}|} \times \frac{\mathbf{v}}{|\mathbf{v}|}$ Flußdichte

$\mathbf{H} = \frac{\mathrm{d}V}{\mathrm{d}\mathbf{s}} = \mu^{-1}\mathbf{B} = \mu_0^{-1}\mathbf{B} - \mathbf{M}$ Feldstärke/Erregung

$\mathbf{M} = n\mathbf{p}_m = \mu_0^{-1}\mathbf{B} - \mathbf{H} = \chi_m \mathbf{H}$ Magnetisierung

$\mathcal{A}$ Vektorpotential

$\mu = \mu_r \mu_0 = (1 + \chi_m)\,\mu_0 = \frac{\mathbf{B}}{\mathbf{H}}$ Permeabilität

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$\mathbf{J} = \frac{\mathrm{d}I}{\mathrm{d}\mathbf{A}} = \rho_V \mathbf{v} = \gamma \mathbf{E}$ $\qquad$ $\mathbf{J}_{\mathrm{mag}} = \nabla \times \mathbf{M}$

$\mathbf{J}_A = \frac{\mathrm{d}I}{\mathrm{d}\mathbf{s}}$ $\qquad$ $\mathbf{J}_{A_{\mathrm{mag}}} = \mathbf{M} \times \frac{\mathbf{A}}{|\mathbf{A}|}$

$\mathbf{p}_m = \pi a^2 \mathbf{e}_n$

$V = \int \mathbf{J} \cdot \mathrm{d}\mathbf{A} = \oint \mathbf{H} \cdot \mathrm{d}\mathbf{s} = NI = \frac{\phi}{Y_m}$ Durchflutung

$\phi = \int \mathbf{B} \cdot \mathrm{d}\mathbf{A} = \oint \mathcal{A} \cdot \mathrm{d}\mathbf{s} = \frac{1}{N}\int U \mathrm{d}t = L\frac{I}{N} = VY_m$ Fluß

$\psi = N\phi = -\int U \mathrm{d}t = LI$ verketteter Fluß

$L = \frac{\psi}{I} = N^2 Y_m = 2\frac{W}{I^2}$ $\qquad$ $M = \frac{\mu}{4\pi}\iint \frac{1}{r}\mathrm{d}\mathbf{s}_1 \mathrm{d}\mathbf{s}_2$

$Y_m = \frac{1}{Z_m} = \frac{\phi}{V} = \mu\frac{A}{d}$ magnetischer Leitwert

Beziehungen mit ε, γ, μ

$\mathbf{D} = \varepsilon\mathbf{E}$ $\quad \mathbf{J} = \gamma\mathbf{E}$ $\quad \mathbf{B} = \mu\mathbf{H}$ $\quad \mathbf{c} = \frac{1}{\sqrt{\varepsilon\mu}}\frac{\mathbf{k}}{|\mathbf{k}|}$ $\quad Y = \sqrt{\frac{\varepsilon}{\mu}} = \left[\frac{H}{E}\right]_{\text{ebene Welle}}$ $\quad$ Relaxationszeit: $T_r = \frac{\varepsilon}{\gamma}$

Wellengleichung ($\varepsilon, \gamma, \mu = \text{const}$) (gilt für $\mathbf{E}, \mathbf{D}, \mathbf{J}, \mathbf{H}, \mathbf{B}$)

$\nabla^2\mathbf{E} - \nabla\cdot(\nabla\cdot\mathbf{E}) = \gamma\mu\frac{\partial\mathbf{E}}{\partial t} + \varepsilon\mu\frac{\partial^2\mathbf{E}}{\partial t^2}$ $\quad \mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 \exp\left(\mathrm{i}\left(\mathbf{k}\cdot\mathbf{r} - \omega t\right)\right)$

Wellenzahl: $\mathbf{k}^2 = \omega^2\mu\varepsilon\left(1 - \mathrm{i}\frac{\gamma}{\omega\varepsilon}\right)$ $\quad$ Dämpfungskonstante: $\Im(k)$ $\quad$ Phasenkonstante: $\Re(k) = \frac{2\pi}{\lambda}$

Energiestrom $\quad \mathbf{S} = \frac{\mathrm{d}P}{\mathrm{d}\mathbf{A}} = \mathbf{E}\times\mathbf{H} = \mathbf{c}w_V$ $\quad w_V = w_{V_e} + w_{V_m}$ $\quad w_{V_e} = w_{V_m}$

Kugelwelle

$$\phi = \frac{\mathbf{r}\cdot\mathbf{p}_e}{4\pi\varepsilon r^3}\left(1 - \mathrm{i}\mathbf{k}\cdot\mathbf{r}\right)\exp\left(\mathrm{i}\left(\mathbf{k}\cdot\mathbf{r} - \omega t\right)\right)$$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon r^5}\left(\mathbf{r}\left(\mathbf{r}\cdot\mathbf{p}_e\right)\left(3 - \mathrm{i}3\mathbf{k}\cdot\mathbf{r} - \left(\mathbf{k}\cdot\mathbf{r}\right)^2\right) - r^2\mathbf{p}_e\left(1 - \mathrm{i}\mathbf{k}\cdot\mathbf{r} - \left(\mathbf{k}\cdot\mathbf{r}\right)^2\right)\right)\exp\left(\mathrm{i}\left(\mathbf{k}\cdot\mathbf{r} - \omega t\right)\right)$$

$$\mathcal{A} = -\mathrm{i}\frac{\mu\omega\mathbf{p}_e}{4\pi r}\exp\left(\mathrm{i}\left(\mathbf{k}\cdot\mathbf{r} - \omega t\right)\right)$$

$$\mathbf{H} = -\mathrm{i}\frac{\left(\mathbf{p}_e\times\mathbf{r}\right)}{4\pi r^3}\left(1 - i\mathbf{k}\cdot\mathbf{r}\right)\exp\left(\mathrm{i}\left(\mathbf{k}\cdot\mathbf{r} - \omega t\right)\right)$$

Potentialdifferentialgleichungen

Eichung		
$\nabla\cdot\mathcal{A} = 0$	$\nabla^2\phi = -\frac{\rho_V}{\varepsilon}$	$\nabla^2\mathcal{A} = -\mu\mathbf{J}$
$\nabla\cdot\mathcal{A} = -\gamma\mu\phi$	$\nabla^2\phi = \gamma\mu\dot{\phi} - \frac{\rho_V}{\varepsilon}$	$\nabla^2\mathcal{A} = \gamma\mu\dot{\mathcal{A}}$
$\nabla\cdot\mathcal{A} = -\varepsilon\mu\dot{\phi}$	$\nabla^2\phi = \varepsilon\mu\ddot{\phi} - \frac{\rho_V}{\varepsilon}$	$\nabla^2\mathcal{A} = \varepsilon\mu\ddot{\mathcal{A}} - \mu\mathbf{J}$

Randbedingungen

Dielektrika	Leiter	Ferromagnetika
$E_{t1} = E_{t2}$	$E_{t1} = E_{t2}$	$H_{t1} = H_{t2} + J_F$
$D_{n1} = D_{n2} + \rho_F$	$J_{n1} = J_{n2}$	$B_{n1} = B_{n2}$
$\frac{\tan(\alpha_1)}{\tan(\alpha_2)} = \frac{E_{n2}}{E_{n1}} = \frac{D_{t1}}{D_{t2}} = \left(1 - \frac{\rho_F}{D_{n1}}\right)\frac{\varepsilon_1}{\varepsilon_2}$	$\frac{\tan(\alpha_1)}{\tan(\alpha_2)} = \frac{E_{n2}}{E_{n1}} = \frac{J_{t1}}{J_{t2}} = \frac{\gamma_1}{\gamma_2}$	$\frac{\tan(\alpha_1)}{\tan(\alpha_2)} = \frac{H_{n2}}{H_{n1}} = \frac{B_{t1}}{B_{t2}} = \left(1 + \frac{J_F}{H_{t2}}\right)\frac{\mu_1}{\mu_2}$

Felder ($\frac{q}{\varepsilon} \mathrel{\widehat{=}} \frac{I}{\gamma}$)

$$\phi = \int\frac{1}{4\pi\varepsilon}\rho_V\left(\mathbf{r}'\right)\left(\frac{1}{r} - \frac{1}{r_0}\right)\mathrm{d}V' = \frac{1}{4\pi\varepsilon}Q\left(\frac{1}{r} - \frac{1}{r_0}\right) = \frac{1}{2\pi\varepsilon}\rho_L\ln\left(\frac{r_0}{r}\right) = \frac{1}{2\varepsilon}\rho_A\left(r_0 - r\right) = \frac{1}{4\pi\varepsilon}\mathbf{p}_e\cdot\left(\frac{\mathbf{r}}{r^3} - \frac{\mathbf{r}_0}{r_0^3}\right)$$

$$\mathbf{E} = \int\frac{1}{4\pi\varepsilon}\rho_V\left(\mathbf{r}'\right)\frac{\mathbf{r}}{r^3}\mathrm{d}V' = \frac{1}{4\pi\varepsilon}Q\frac{\mathbf{r}}{r^3} = \frac{1}{2\pi\varepsilon}\rho_L\frac{\mathbf{r}}{r^2} = \frac{1}{2\varepsilon}\rho_A\frac{\mathbf{r}}{r} = \frac{1}{4\pi\varepsilon}p_e\frac{1}{r^3}\left(2\cos\left(\vartheta\right)\mathbf{e}_r + \sin\left(\vartheta\right)\mathbf{e}_\phi\right)$$

$$\mathcal{A} = \int\frac{\mu}{4\pi}\mathbf{J}\left(\mathbf{r}'\right)\left(\frac{1}{r} - \frac{1}{r_0}\right)\mathrm{d}V' = \int\frac{\mu}{3\pi}\mathbf{I}\left(\mathbf{r}'\right)\left(\frac{1}{r} - \frac{1}{r_0}\right)\mathrm{d}s' = \frac{\mu}{2\pi}\mathbf{I}_L\ln\left(\frac{r_0}{r}\right) = \frac{\mu}{2}\mathbf{I}_A\left(r_0 - r\right) = \frac{\mu}{4\pi}\mathbf{p}_m\times\left(\frac{\mathbf{r}}{r^3} - \frac{\mathbf{r}_0}{r_0^3}\right)$$

$$\mathbf{B} = \int\frac{\mu}{4\pi}\mathbf{J}\left(\mathbf{r}'\right)\times\frac{\mathbf{r}}{r^3}\mathrm{d}V' = \int\frac{\mu}{4\pi}\mathbf{I}\left(\mathbf{r}'\right)\times\frac{\mathbf{r}}{r^3}\mathrm{d}s' = \frac{\mu}{2\pi}\mathbf{I}_L\times\frac{\mathbf{r}}{r^2} = \frac{\mu}{2}\mathbf{I}_A\times\frac{\mathbf{r}}{r} = \frac{\mu}{4\pi}p_m\frac{1}{r^3}\left(2\cos\left(\vartheta\right)\mathbf{e}_r + \sin\left(\vartheta\right)\mathbf{e}_\phi\right) = \frac{\mu}{4\pi}Q\mathbf{v}\times\frac{\mathbf{r}}{r^3}$$